PENETRATION OF THIN IONOSPHERIC LAYERS *

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ABSTRACT. Reflection coefficient of a thin ionospheric region, having parabolic ionisation gradient, has been deduced by adapting the B. W. K. method of computation as applied in the case of a parabolic potential barrier. It is found that the reflection co-efficient of such a region is ·5 and not zero at the so-called critical frequency.

Two curves, one for the E- and the other for the F-region have been drawn depicting the variation of reflection coefficient with frequency It is found that the transition from complete reflection to complete penetration takes place more suddenly for Region F than for Region E Complete penetration is effected when $\Delta f/\sqrt{f_0}$ is approximately 50 for Region E and 30 for Region F. (f_0 critical frequency , Δf departure of wave frequency from f_0).

INTRODUCTION

According to the Eccles-Larmor formula the refractive index μ of an ionised gas is given by

$$\mu^2 = 1 - \frac{4\pi e^2}{mp^2} N \qquad \dots \quad (1)$$

where N is the electron density and p the pulsatance of the wave.

It follows that a wave of frequency f incident vertically on an ionospheric region of maximum density N_{max} will either be transmitted or totally reflected by the layer according as f is greater or less than

$$f_0 = \left[\frac{N_{max} e^2}{\pi m} \right]^{\frac{1}{2}} . \qquad \dots \quad (2)$$

Now in actual experiment it is found that the same wave is reflected both from Regions E and F. This means that there is not total but partial reflection from Region E for frequencies not far removed from the critical frequency. This anomalous result can be explained if it is remembered that the ray treatment of geometrical optics—after which the reflection and penetration condition (2) has been obtained—ceases to be valid when f approaches f_0. This is because [1] when $\mu \to 0$ the wavelength $\lambda \to \infty$ and, it is well known that simple treatment of geometrical optics fails when the refractive index of a medium varies appreciably in the distance of a wavelength. The case of the penetration

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of an ionospheric region, by exploring waves of frequencies near the critical frequency f_0, has therefore to be treated according to wave optics rather than according to geometrical optics. This has been done by several workers[2] and among the recent ones we may mention the work of Saha and Rai. These authors have adapted the famous treatment of Gamow[3] on calculation of the transparency of the potential barrier surrounding an atomic nucleus to the problem of propagation of electromagnetic waves through an ionospheric "electron-barrier." For numerical calculations they have assumed a linear gradient (the outline of the barrier being in the form of an isosceles triangle) and obtained the interesting result that "the amount of energy transmitted falls to half its value for the assumed form of the barrier and for a gradient of electron density characteristic of F layer, if the half breadth of the barrier is about 1·5 km."

Recent observations show that, though the complete form of the electron barrier may not have any simple geometric form, the portion of it near the maximum electron density (which is determinative of the penetration frequency) has the simple form of a parabola. The transparency coefficient of a potential barrier of this form has already been calculated in quantum mechanics by the well-known B. W. K. method and in the present paper this method has been adapted for determining the reflection co-efficients of ionospheric regions having distribution of electron density of similar form.

DISTRIBUTION OF ELECTRON DENSITY

It has been shown by Appleton[4] that according to Chapman's theory of ionised region formation by the action of monochromatic radiation, the electron density N at any level is given by

$$N = N_{max}\left(1 - \frac{Z^2}{4H^2}\right) \qquad \ldots \quad (3)$$

where Z is the distance of the level measured from the level of maximum ionisation and $H = \frac{RT}{Mg}$ the "scale height" for the region. This relation is valid for values of Z up to $Z = H$.

The distribution of N with height is thus parabolic near the level of maximum ionisation.

This conclusion has also been confirmed by experiments of Appleton.[5] He compared experimental $P' - f$ curves with those obtained by theoretical calculation made on the assumption of a parabolic gradient and found that the two curves are in substantial agreement.

CALCULATION OF THE REFLECTION COEFFICIENT FOR A PARABOLIC GRADIENT

The equation of a plane wave in a medium of refractive index μ is given by

$$\frac{d^2\psi}{dz^2} = -\left(\frac{2\pi}{\lambda}\right)^2 \mu^2\psi$$

where λ is the wavelength in vacuum.

For an ionised layer, the refractive index of a wave of pulsatance p is given by equation (1).

If the ionosphere is horizontally stratified and the z-axis is taken in the upward direction then the equation for vertical propagation becomes

$$\frac{d^2\psi}{dz^2} = -\left(\frac{2\pi}{\lambda}\right)^2\left(1 - \frac{4\pi c^2 N}{mp^2}\right)\psi.$$

The wave-length in the medium is no longer constant and varies as N varies being equal to $\frac{\lambda_{vac}}{\mu}$. Putting $u=\frac{zp}{c}$ and $p_0^2=\frac{4\pi c^2 N}{m}$, we have

$$\frac{d^2\psi}{du^2} + (1 - p_0^2/p^2)\psi = 0. \qquad \ldots \quad (4)$$

The above equation is for a field-free space. If there be a magnetic field, the equation is profoundly modified. The refractive index becomes complex and is given by the well-known Appleton-Hartree formula. In this case the action of the terrestrial magnetic field may be eliminated by a suitable choice of the direction of propagation of the wave. If the propagation be in a vertical plane in the magnetic equator and the direction of the electric vector of the wave is parallel to the magnetic lines of force then the oscillations of the electrons and ions due to the passage of the wave are not affected. Equation (4) can therefore be applied for the ionospheric exploration by vertical propagation near the magnetic equator when the wave is plane-polarised with the direction of the electric vector north-south.

Now, making the substitutions

$$N = mp_0^2/4\pi e^2$$

$$N_{max} = \frac{mp_{max}^2}{4\pi e^2}$$

and

$$z = cu/p,$$

in equation (3), we derive the relation

$$\frac{p_0^2}{p^2}=\frac{p_{max}^2}{p^2}-\frac{\pi^2 f_0^2 c^2}{H^2 p^4}u^2$$

$$=\frac{p_{max}^2}{p^2}-a^2u^2 \qquad \dots (5)$$

which also is an equation of parabola corresponding to equation (3) with $H^2p^4/\pi^2 f_0^2 c^2$ as the latus-rectum.

Substituting the value of p_0^2/p^2 in equation (4) we get the relation

$$\frac{d^2\psi}{du^2}+(\beta+a^2u^2)\psi=0,$$

where $\beta=1-p_{max}^2/p^2$.

The solution of this differential equation giving the equation of reflected and transmitted wave has already been obtained by Kemble with the help of the B. W. K. approximation functions and Krammer's connection formula for the passage of ψ-waves through a potential barrier of parabolic form. Corresponding to the dependence, in quantum mechanics, of the refractive index of the ψ-waves upon the potential, we have the dependence of the same, for the radio-waves in ionosphere, upon the electron density N The potential barrier for ψ-wave thus corresponds to "electron-barrier" for radio-waves.

Without going through the calculations we can therefore write out at once, after Kemble,[6] the expression for T, the transmission coefficient of the radio-waves through the electron-barrier.

For $f<f_0$

$$T=\frac{1}{1+e^{2K}}, \qquad \dots (6)$$

where $$K=\int_{u_1}^{u_2}\sqrt{1-p_0^2/p^2}\,d\xi \qquad \dots (7)$$

and $$\xi=\sqrt{a}\,u.$$

The limits u_1 and u_2 are the values of u where the line $p_0^2/p^2=1$ cuts the electron-barrier. The significance of these limits will be readily understood from a reference to Fig. 1. u_1 represents the distance measured downwards in the

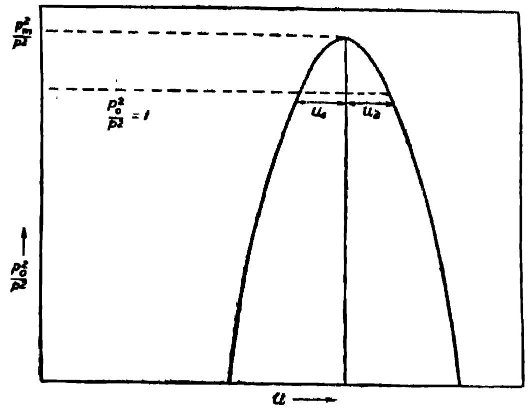

FIGURE 1

ionised layer from the level of maximum ionisation, to the point to which the wave of frequency p would penetrate according to the ray treatment. In other words, u_1 marks the position of the level at which total reflection ought to take place. The wave treatment shows, however, that a fraction of the wave will be transmitted beyond this level and this fraction is given by equation (6). In order to evalute T we have to find the value of the integral (7). If we take the point where the line $p_0^2/p^2=1$ cuts the axis of the parabola as origin, equation (5) may be written as

$$1-p_0^2/p^2=\alpha^2u^2.$$

Hence
$$K=\alpha^{\frac{3}{2}}\,2\int_0^{u_0} u\,du,$$

where $u_0=u_1=u_2$ is the half-breadth of the barrier. Or, since u_0^2 is equal to $\frac{\frac{p_m^2}{p^2}-1}{\alpha^2}$,

$$K=\alpha^{-\frac{1}{2}}\frac{p_m^2-p^2}{\alpha^2}$$

$$=2\pi\left(\frac{H}{\pi c}\right)^{\frac{1}{2}}\frac{1}{\sqrt{f_0}}\quad\frac{f_0^2-f^2}{f}$$

$$=4\cdot\left[\frac{\pi H}{c}\right]^{\frac{1}{2}}\frac{\Delta f}{\sqrt{f_0}}.$$

For any particular value of H, K can now be calculated for various values of $\Delta f/\sqrt{f_0}$ and the transmission coefficient determined therefrom. Fig. 2 shows

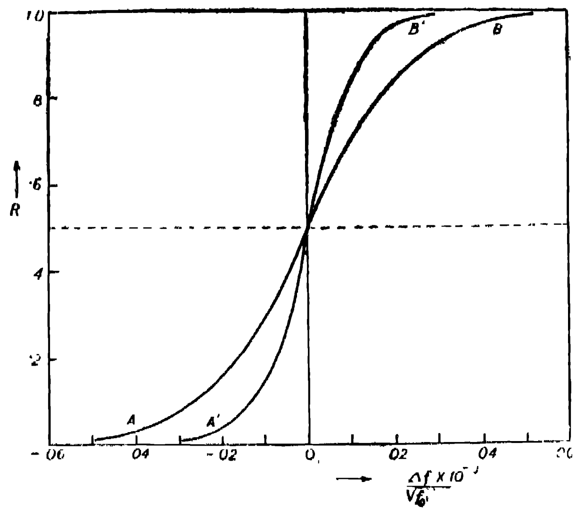

FIGURE 2

how reflection coefficient $R(=1-T)$ varies with the departure of the exploring frequency from the critical frequency f_0. The values of H^7 assumed for the calculation, are 11·4 km. for E-region (curve AB) and 50 km. for the F-region (curve A'B'). It shows how the reflection coefficient varies when the frequency of the exploring waves is decreased below the frequency of penetration.

For $f > f_0$, that is, for the case when the exploring wave frequency is *increased above* the penetration frequency, we are required to find out the value of u_0 when $p^2 > p_m^2$. In this case the line $p_0^2/p^2 = 1$ cuts the parabola in two imaginary points. u_0 being then imaginary, u_0^2 is real and negative and so also $K = u_0^2 a^{\frac{3}{2}}$. Instead of (6) we now have

$$T = \frac{1}{1+e^{-2k}},$$

and the portion of the curve in Fig. 2, depicting the variation of $\Delta f/\sqrt{f_0}$ with R for the case $f > f_0$, can therefore be obtained by simply plotting R for Δf negative.

CONCLUSION

From the curves in Fig. 2 it is seen that the reflection coefficient is not zero for the so-called critical frequency. For the two cases considered it becomes so only when the frequency has been increased above the critical value by about 50 and 30 times of the square-roots of the critical frequencies. This shows that the reading of critical frequency from the $P'-f$ curve in the usual way, *i.e.*, by noting the frequency at which reflection from the ionised region ceases, entails a certain percentage of error.

A comparison of the two curves, for $H = 11.4$ km. and for $H = 50$ km. representing E- and F-regions respectively, shows that for a region which is more diffuse, a smaller percentage departure from f_0 is needed for complete penetration. In other words, for a diffuse region, the transition from complete reflection to complete transmission is sharper.

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